# Definite Integration

## Question1

**The value of $\int_{-1}^{1} \sin^5 x \cos^4 x dx$ is**

### KCET 2025

**Options:**

A. $-\pi/2$

B. $\pi$

C. $\pi/2$

D. 0

**Answer: D**

### Solution:

$$\int_{-1}^{1} \sin^5 x \cos^4 x dx = 0$$

Since it is odd function.

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## Question2

The value of $\int_0^{2\pi} \sqrt{1+\sin\left(\frac{x}{2}\right)} dx$ is

### KCET 2025

**Options:**

A. 8

B. 4

C. 2

D. 2

**Answer: A**

**Solution:**

$$\int_0^{2\pi} \sqrt{1+\sin\frac{x}{2}}$$
$$\int_0^{2\pi} \left|\cos\frac{x}{4}+\sin\frac{x}{4}\right| = \left[4\sin\frac{x}{4}-4\cos\frac{x}{4}\right]_0^{2\pi}$$
$$= (4(1)-0(0-4)$$
$$= 8$$

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# Question3

$\int_0^1 \log\left(\frac{1}{x}-1\right)dx$ **is**

### KCET 2025

**Options:**

A. 1

B. 0

C. $\log_e 2$

D. $\log_e\left(\frac{1}{2}\right)$

**Answer: B**

**Solution:**

$$\int_0^1 \log\left(\frac{1}{x}-1\right)dx = \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$
$$I = \int_0^1 \log\left(\frac{1}{1+0-x}-1\right) = \int_0^1 \log\left(\frac{1}{1-x}-1\right)dx$$
$$I = \int_0^1 \log\left(\frac{1-1+x}{1-x}\right)dx = \int_0^1 \log\left(\frac{x}{1-x}\right)dx$$
$$2I = \int_0^1 \left(\log\left(\frac{1-x}{x}\right)+\log\left(\frac{x}{1-x}\right)\right)dx$$
$$2I = 0 \quad \Rightarrow I = 0.$$

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# Question4

$\int_{-\pi}^{\pi} \left(1-x^2\right)\sin x \cdot \cos^2 x dx$ **is**

### KCET 2024

**Options:**

A. $\pi-\frac{\pi^2}{3}$

B. $2\pi-\pi^3$

C. $\pi-\frac{\pi^3}{2}$

D. 0

**Answer: D**

### Solution:

$$\begin{aligned}\because \text{ Let } f(x) &= (1-x^2)\sin x\cdot\cos^2 x\\ f(-x) &= (1-(-x)^2)\sin(-x)\cdot\cos^2(-x)\\ &= (1-x^2)(-\sin x)(\cos^2 x)\\ &= -(1-x^2)(\sin x)(\cos^2 x)\end{aligned}$$

So, $\quad f(x)=-f(-x)$

So, $f(x)$ is an odd function.

Hence, $\int_{-\pi}^{\pi}(1-x^2)\sin x\cdot\cos^2 x dx=0$

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# Question5

$$\int_1^5(|x-3|+|1-x|)dx=$$

### KCET 2024

**Options:**

A. 12

B. 5/6

C. 21

D. 10

**Answer: A**

### Solution:

$$\int_1^5 [| x-3| + |1-x|] dx$$
$$= \int_1^5 |x-3| dx + \int_1^5 |1-x| dx$$
$$= \int_1^3 |x-3| dx + \int_3^5 |x-3| dx + \int_1^5 |1-x| dx$$
$$= \int_1^3 (3-x) dx + \int_3^5 (x-3) dx + \int_1^5 (x-1) dx$$
$$= \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^5 + \left[\frac{x^2}{2} - x\right]_1^5$$
$$= \left(3 \times 3 - \frac{9}{2}\right)^{-} - \left(3 \times 1 - \frac{1}{2}\right) + \left(\frac{5 \times 5}{2} - 3 \times 5\right)$$
$$- \left(\frac{3 \times 3}{2} - 3 \times 3\right) + \left(\frac{5 \times 5}{2} - 5\right) - \left(\frac{1}{2} - 1\right)$$
$$= \frac{9}{2} - \frac{5}{2} - \frac{5}{2} + \frac{9}{2} + \frac{15}{2} + \frac{1}{2}$$
$$= \frac{9 - 5 - 5 + 9 + 15 + 1}{2} = \frac{24}{2} = 12$$

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# Question6

$\int\limits_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}} dx$ is equals to :

## KCET 2023

**Options:**

A. 6

B. 4

C. 3

D. 5

**Answer: C**

## Solution:

Let $I = \int\limits_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}} dx$ ..... (i)

Using property $\int\limits_0^a f(x) dx = \int_0^a f(0 + a - x) dx$

$$I = \int_2^8 5^{\sqrt{10-(2+8-x)}}$$

$$I = \int_2^8 \frac{5^{\sqrt{10-10+x}}}{5^{\sqrt{10-x}} + 5^{\sqrt{10-10+x}}}$$

$$I = \int_2^8 \frac{5^{\sqrt{x}}}{5^{\sqrt{10-x}} + 5^{\sqrt{x}}} \quad \text{.... (ii)}$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_2^8 \frac{5^{\sqrt{10-x}} + 5^{\sqrt{x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}} dx$$

$$2I = \int_2^8 dx$$

$$I = \frac{1}{2}[x]_2^8 = \frac{1}{2} \times 6 = 3$$

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# Question7

**$\int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right) dx$ is equals to**

### KCET 2023

**Options:**

A. 3

B. 4

C. 1

D. 0

**Answer: B**

## Solution:

Let

$$I = \int_{-2}^{0} x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1) dx$$

$$I = \int_{-2}^{0} \left[(x+1)^3 + 2 + (x+1)\cos(x+1) dx\right.$$

Putting, $x + 1 = t$.

Then $dx = dt$, we get

$$I = \int_{-1}^{1} [t^3 + 2 + t\cos t]\, dt$$

$$I = \int_{-1}^{1} (2)u_{.} = [2t]_{-1}^{1} = 2[1 - (-1)] = 4$$

As $t^3 + t\cos t$ is an odd function.

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# Question8

$\int_0^\pi \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x} dx$ **is equals to**

## KCET 2023

**Options:**

A. $\pi^2/4$

B. $\pi/2$

C. $\pi^2/2$

D. $\pi/4$

**Answer: A**

## Solution:

Let $I = \int_0^\pi \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x} dx$ .... (i)

Using property $I = \int_0^\pi f(x) dx$

$= \int_0^\pi f(0 + \pi - x) dx.$

Then, $I = \int_0^\pi \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x)\operatorname{cosec}(\pi - x)} dx$

$I = \int_0^\pi \frac{(\pi - x)\tan x}{\sec x \operatorname{cosec} x} dx$ ..... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^\pi \frac{\tan x(x + \pi - x)}{\sec x \cdot \operatorname{cosec} x} dx$$

$$2I = \int_0^\pi \frac{\pi \tan x}{\sec x \operatorname{cosec} x} dx$$

$$2I = \pi \int_0^\pi \sin^2 x dx$$

$$2I = \pi \int_0^\pi \frac{1 - \cos 2x}{2} dx$$

$$2I = \pi \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^\pi$$

$$2I = \pi \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi^2}{4}$$

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# Question9

**If $[x]$ is the greatest integer function not greater than $x$, then $\int_0^8 [x] dx$ is equal to**

## KCET 2022

**Options:**

A. 28

B. 30

C. 29

D. 20

**Answer: A**

### Solution:

Let $I = \int_0^8 [x]dx$

$$I = \int_0^1 [x]dx + \int_1^2 [x]dx + \int_2^3 [x]dx + \int_3^4 [x]dx + \int_4^5 [x]dx + \int_5^6 [x]dx + \int_6^7 [x]dx + \int_7^8 [x]dx$$

$$= \int_0^1 0dx + \int_1^2 1dx + \int_2^3 2dx + \int_3^4 3dx + \int_4^5 4dx + \int_5^6 5dx + \int_6^7 6dx + \int_7^8 7dx$$

$$= 0 + 1[x]_1^2 + 2[x]_2^3 + 3[x]_3^4 + 4[x]_4^5 + 5[x]_5^6 + 6[x]_6^7 + 7[x]_7^8$$

$$= 0 + 1(2-1) + 2(3-2) + 3(4-3) + 4(5-4) + 5(6-5) + 6(7-6) + 7(8-7)$$

$$= 1 + 2 + 3 + 4 + 5 + 6 + 7$$

$$= \frac{7 \cdot (7+1)}{2} = \frac{7 \cdot 8}{2} = 7 \cdot 4 = 28$$

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# Question10

**$\int_0^{\pi/2} \sqrt{\sin\theta} \cos^3\theta d\theta$ is equal to**

## KCET 2022

**Options:**

A. $\frac{8}{23}$

B. $\frac{7}{23}$

C. $\frac{8}{21}$

D. $\frac{7}{21}$

**Answer: C**

## Solution:

Let $I = \int_0^{\pi/2} \sqrt{\sin\theta} \cdot \cos^3\theta d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

When $\theta = 0 \Rightarrow t = 0$

When $\theta = \frac{\pi}{2} \Rightarrow t = 1$

$$I = \int_0^1 \sqrt{t}\left(1 - t^2\right)dt = \int_0^1 \left(t^{1/2} - t^{5/2}\right) \cdot dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 - \left[\frac{\frac{7}{2}}{\frac{7}{2}}\right]_0^1 = \frac{2}{3}[1-0] - \frac{2}{7}[1-0]$$

$$= \frac{2}{3} - \frac{2}{7} = \frac{14-6}{21} = \frac{8}{21}$$

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# Question11

$\int_0^1 \frac{xe^x}{(2+x)^3} dx$ **is equal to**

### KCET 2022

**Options:**

A. $\frac{1}{27} \cdot e - \frac{1}{8}$

B. $\frac{1}{27} \cdot e + \frac{1}{8}$

C. $\frac{1}{9} \cdot e + \frac{1}{4}$

D. $\frac{1}{9} \cdot e - \frac{1}{4}$

**Answer: D**

### Solution:

$$\text{Let } I = \int_0^1 \frac{xe^x}{(2+x)^3} dx = \int_0^1 e^x \left[\frac{(2+x)-2}{(2+x)^3}\right] dx$$

$$= \int_0^1 e^x \left[\frac{1}{(2+x)^2} - \frac{2}{(2+x)^3}\right] dx$$

Here, $\frac{1}{(2+x)^2} = f(x)$ and $\frac{-2}{(2+x)^3} = f'(x)$

We know, $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$

$I = \left[e^x \cdot \frac{1}{(2+x)^2}\right]_0^1 = \frac{e^x}{(2+1)^2} - \frac{e^0}{(2+0)^2} = \frac{e}{9} - \frac{1}{4}$

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# Question12

**Evaluate** $\int_2^3 x^2 dx$ **as the limit of a sum**

### KCET 2022

**Options:**

A. $\frac{72}{6}$

B. $\frac{53}{9}$

C. $\frac{25}{7}$

D. $\frac{19}{3}$

**Answer: D**

## Solution:

Let $I=\int_{2}^{3} x^{2} d x$

$I=\int_{2}^{3} x^{2} d x=\int_{2}^{3} f(x) d x$, where $f(x)=x^{2}$

Also, $a=2, b=3$

$$\lim_{h \rightarrow 0} h[f(2)+f(2+h)+f(2+2h) +\ldots+f(2+(n-1)h)]$$

where, $nh=b-a=3-2=1$

$$=\lim_{h \rightarrow 0} h\left[2^{2}+(2+h)^{2}+(2+2h)^{2}+\ldots .+(2+(n-1)h)^{2}\right]$$

$$=\lim_{h \rightarrow 0} h\left[2^{2}+\left(2^{2}+h^{2}+4h\right)+\left(2^{2}+2^{2}h^{2}+8h\right) +\ldots+\left(2^{2}+(n-1)^{2}h^{2}+4(n-1)h\right)\right]$$

$$=\lim_{h \rightarrow 0} h\left[2^{2}h+h^{2}\left(1^{2}+2^{2}+\ldots . .+(n-1)^{2}\right) +4h(1+2+\ldots+(n-1))\right]$$

$$=\lim_{h \rightarrow 0}\left[4nh+\frac{h^{3}(n-1)n(2(n-1)+1)}{6}+4h^{2} \frac{(n-1)h}{2}\right]$$

$$=\lim_{h \rightarrow 0}\left[4nh+\frac{(nh-h)(nh)(2nh-h)}{6}+2(nh-h)(nh)\right]$$

$$=\lim_{h \rightarrow 0}\left[4 \times 1+\frac{(1-h) \cdot 1(2-h)}{6}+2(1-h) \cdot 1\right]$$

$$=4+\frac{2}{6}+2=6+\frac{1}{3}=\frac{19}{3}$$

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# Question13

$\int_{0}^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$ **is equal to**

## KCET 2022

**Options:**

A. $\log 2-1$

B. $\log 2$

C. $-\log 2$

D. $1-\log 2$

**Answer: D**

## Solution:

Let $I = \int\limits_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$

$= \int\limits_0^{\pi/2} \frac{\cos x(1+\sin x - 1)}{1+\sin x} dx$

$I = \int\limits_0^{\pi/2} \left( \frac{\cos x(1+\sin x)}{1+\sin x} - \frac{\cos x}{1+\sin x} \right) dx$

$= \int\limits_0^{\pi/2} \cos x dx - \int\limits_0^{\pi/2} \frac{\cos x}{1+\sin x} dx = [\sin x]_0^{\pi/2} - I_1$ .... (i)

$I_1 = \int\limits_0^{\pi/2} \frac{\cos x}{1+\sin x} dx$

Let $1 + \sin x = t$

$\Rightarrow \cos x dx = dt$

$I_1 = \int\limits_0^{\pi/2} \frac{dt}{t} = [\log t]_0^{\pi/2} = [\log(1+\sin x)]_0^{\pi/2}$

$= \log(1+1) - \log(1+0) = \log 2$

Now, from Eq. (i), we have

$I = [1-0] - \log 2 = 1 - \log 2$

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# Question14

**If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$, where $n$ is positive integer, then $I_{10} + I_8$ is equal to**

## KCET 2021

**Options:**

A. 9

B. $\frac{1}{7}$

C. $\frac{1}{8}$

D. $\frac{1}{9}$

**Answer: D**

## Solution:

$I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$ .... (i)

$I_{n+2} = \int_0^{\frac{\pi}{4}} \tan^{n+2} x dx$ .... (ii)

Adding Eqs. (i) and (ii),

$$I_n + I_{n+2} = \int_0^{\frac{\pi}{4}} \tan^n x dx + \int_0^{\frac{\pi}{4}} \tan^{n+2} x dx$$
$$= \int_0^{\frac{\pi}{4}} \tan^n x \cdot \sec^2 x dx$$
$$= \left[\frac{\tan^{n+1} x}{n+1}\right]_0^{\frac{\pi}{4}} = \frac{1}{n+1}$$

Thus, $I_n + I_{n+2} = \frac{1}{(n+1)}$

Substitute 8 for $x$.

$I_8 + I_{10} = \frac{1}{9}$

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# Question15

**The value of $\int_0^{4042} \frac{\sqrt{x}dx}{\sqrt{x}+\sqrt{4042-x}}$ is equal to**

## KCET 2021

**Options:**

A. 4042

B. 2021

C. 8084

D. 1010

**Answer: B**

## Solution:

Let $I = \int_0^{4042} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4042-x}} dx$ .... (i)

Using the property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{4042} \frac{\sqrt{4042-x}}{\sqrt{4042-x}+\sqrt{x}}$ ..... (ii)

Adding Eqs. (i) and (ii),

$$2I = \int_0^{4042} \frac{\sqrt{x}+\sqrt{4042-x}}{\sqrt{4042-x}+\sqrt{x}} dx$$
$$\Rightarrow \quad 2I = \int_0^{4042} I dx$$
$$\Rightarrow \quad 2I = 4042$$
$$\Rightarrow \quad I = 2021$$

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# Question16

**The value of $\int_{-1/2}^{1/2} \cos^{-1} x dx$ is**

### KCET 2020

**Options:**

A. $\pi$

B. $\frac{\pi}{2}$

C. 1

D. $\frac{\pi^2}{2}$

**Answer: B**

### Solution:

We have, $\int_{-1/2}^{1/2} \cos^{-1} x dx$

$= \int_{-1/2}^{1/2} (\pi/2 - \sin^{-1} x) dx$

$$= \int_{-1/2}^{1/2} \pi/2 dx - \int_{-1/2}^{1/2} \sin^{-1} x dx$$
$$= \frac{\pi}{2} \int_{-1/2}^{1/2} dx - 0 \quad (\because \sin^{-1} x \text{ is odd function })$$
$$= \frac{\pi}{2}[x]_{-1/2}^{1/2} = \frac{\pi}{2}\left[\frac{1}{2} - (-1/2)\right] = \frac{\pi}{2}(1) = \frac{\pi}{2}$$

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# Question17

**Find the value of $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ is**

### KCET 2020

**Options:**

A. $\frac{\pi}{2} \log 2$

B. $\frac{\pi}{4} \log 2$

C. $\frac{1}{2}$

D. $\frac{\pi}{8} \log 2$

**Answer: D**

### Solution:

Let $I=\int_0^1 \frac{\log(1+x)}{1+x^2}dx$

Put $\quad x=\tan\theta$

$\Rightarrow \quad dx=\sec^2\theta d\theta$

When $x=0, \theta=0$

and when $x=1, \theta=\pi/4$

$$\therefore \quad \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{1+\tan^2\theta}\left(\sec^2\theta\right)d\theta$$

$$I=\int_0^{\pi/4} \log(1+\tan\theta)d\theta \quad \text{.... (i)}$$

$$=\int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$=\int_0^{\pi/4} \log\left[1+\frac{\tan\pi/4-\tan\theta}{1+\tan\pi/4\tan\theta}\right]d\theta$$

$$=\int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$$

$$=\int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$$

$\int_0^{\pi/4}[\log 2-\log 1+\tan\theta)]d\theta$ .... (ii)

On adding Eqs. (i) and (ii), we get

$$2I=\int_0^{\pi/4} \log 2d\theta$$
$$=\log 2\int_0^{\pi/4} 1d\theta=\log(\theta)_0^{\pi/4}$$
$$=\log(\pi/4-0)=\frac{\pi}{4}\log 2$$
$$\Rightarrow \quad I=\frac{\pi}{8}\log 2$$

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# Question18

**The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx$ is**

### KCET 2020

**Options:**

A. 2

B. 0

C. 1

D. $-2$

**Answer: C**

### Solution:

We have, $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

$$= \int_0^{\pi/2} \left( \frac{\cos x}{1+e^x} + \frac{\cos(-x)}{(1+e^{-x})} \right) dx$$
$$\left[ \because \int_{-a}^{a} f(x)dx = \int_0^a (f(x) + f(-x))dx \right]$$
$$= \int_0^{\pi/2} \left( \frac{\cos x}{1+e^x} + \frac{e^x \cos x}{1+e^x} \right) dx$$
$$= \int_0^{\pi/2} \cos x \left( \frac{1+e^x}{1+e^x} \right) dx$$
$$= \int_0^{\pi/2} \cos x dx = (\sin x)_0^{\pi/2}$$
$$= \left( \sin \frac{\pi}{2} - \sin 0 \right) = 1$$

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# Question19

$$\int_{-3}^{3} \cot^{-1} x dx =$$

### KCET 2019

**Options:**

A. $6\pi$

B. $3\pi$

C. 3

D. 0

**Answer: B**

## Solution:

$$\int_{-3}^{3} \cot^{-1} x dx = \int_{-3}^{3} \left( \frac{\pi}{2} - \tan^{-1} x \right) dx$$
$$= \int_{-3}^{3} \frac{\pi}{2} dx - \int_{-3}^{3} \tan^{-1} x dx = \frac{\pi}{2} [x]_{-3}^{3} - 0$$
$$(\because \int_{-a}^{a} f(x)dx = 0, \text{ if } f \text{ is odd functions})$$
$$= \frac{\pi}{2}[3+3] = \frac{\pi}{2} \times 6 = 3\pi$$

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# Question20

$$\int_{0}^{2} \left[ x^2 \right] dx =$$

### KCET 2019

**Options:**

A. $5-\sqrt{2}+\sqrt{3}$

B. $5-\sqrt{2}-\sqrt{3}$

C. $-5-\sqrt{2}-\sqrt{3}$

D. $5+\sqrt{2}-\sqrt{3}$

**Answer: B**

### Solution:

$\int_0^2 [x^2]dx = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$

$$= \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$$

$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$
$= (\sqrt{2}-1) + (2\sqrt{3}-2\sqrt{2}) + (6-3\sqrt{3})$
$= 5-\sqrt{2}-\sqrt{3}$

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# Question21

$$\int_0^1 \sqrt{\frac{1+x}{1-x}}dx =$$

### KCET 2019

**Options:**

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2}-1$

C. $\frac{1}{2}$

D. $\frac{\pi}{2}+1$

**Answer: D**

### Solution:

$$\int_0^1 \sqrt{\frac{1+x}{1-x}} dx = \int_0^1 \frac{1+x}{\sqrt{1-x^2}} dx$$
$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx + \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$
$$= \left[\sin^{-1} x\right]_0^1 - \frac{1}{2} \int_0^1 \frac{-2x}{\sqrt{1-x^2}} dx$$
$$= \left(\frac{\pi}{2} - 0\right) - \frac{1}{2} \int_1^0 \frac{dt}{\sqrt{t}}$$

put $1 - x^2 = t \Rightarrow -2x dx = dt$ at $x = 0, t = 1$
and $x = 1, t = 0$

$$= \frac{\pi}{2} + \frac{1}{2} \int_0^1 t^{-1/2} dt$$
$$= \frac{\pi}{2} + \frac{1}{2} \left(\frac{t^{1/2}}{1/2}\right)_0^1 = \frac{\pi}{2} + 1$$

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# Question22

## $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is equal to

## KCET 2018

**Options:**

A. $\frac{\pi}{4} - \tan^{-1}(e)$

B. $\tan^{-1}(e) - \frac{\pi}{4}$

C. $\tan^{-1}(e) + \frac{\pi}{4}$

D. $\tan^{-1}(e)$

**Answer: B**

## Solution:

We have, $I = \int_0^1 \frac{dx}{e^x + e^{-x}}$

$\Rightarrow \quad I = \int_0^1 \frac{e^x}{1 + e^{2x}} dx$

put $e^x = t$ and $e^x dx = dt$, where if
$x = 0, t = 1$ and if $x = 1, t = e$

$\therefore$

$\Rightarrow \quad I = \int_1^e \frac{dt}{1 + t^2}$

$\Rightarrow \quad I = \left[\tan^{-1} t\right]_1^e$

$\Rightarrow \quad I = \tan^{-1} e - \tan^{-1} 1$

$\Rightarrow$

$I = \tan^{-1} e - \frac{\pi}{4}$

Divide by $\cos^2 \theta$, we get

$$I=\int_0^{\pi/6}\frac{\sec^2\theta d\theta}{\sec^2\theta+\tan^2\theta}$$
$$\Rightarrow\quad I=\int_0^{\pi/6}\frac{\sec^2\theta d\theta}{1+2\tan^2\theta}$$
$$\Rightarrow\quad I=\frac{1}{2}\int_0^{\pi/6}\frac{\sec^2\theta d\theta}{\left(\frac{1}{\sqrt{2}}\right)^2+\tan^2\theta}$$

Put $\tan\theta=t,\sec^2\theta d\theta=dt$

$\theta=0,t=0\quad\theta=\frac{\pi}{6},t=\frac{1}{\sqrt{3}}$

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# Question23

$\int_0^{1/2}\frac{dx}{(1+x^2)\sqrt{1-x^2}}$ **is equal to**

## KCET 2018

**Options:**

A. $\frac{1}{\sqrt{2}}\tan^{-1}\sqrt{\frac{2}{3}}$

B. $\frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$

C. $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{3}{2}\right)$

D. $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

**Answer: A**

## Solution:

We have, $I=\int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

Put $x=\sin\theta \Rightarrow dx=\cos\theta d\theta$

$x=0, \theta=0, x=\frac{1}{2}, \theta=\frac{\pi}{6}$

$$\therefore \quad I=\int_0^{\pi/6} \frac{\cos\theta d\theta}{\left(1+\sin^2\theta\right)\cos\theta}$$

$$\therefore \quad I=\int_0^{\pi/6} \frac{d\theta}{1+\sin^2\theta}$$

$$\because \quad I=\frac{1}{2}\int_0^{1/\sqrt{3}} \frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2(+t^2)}$$

$$\Rightarrow \quad I=\frac{1}{2}\left(\sqrt{2}\tan^{-1}\sqrt{2}t\right)_0^{1/\sqrt{3}}$$

$$\Rightarrow \quad I=\frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)-0\right]$$

$$\Rightarrow \quad I=\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

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## Question24

$\int_{-2}^{2}|x\cos\pi x|dx$ **is equal to**

**KCET 2018**

**Options:**

A. $\frac{8}{\pi}$

B. $\frac{4}{\pi}$

C. $\frac{2}{\pi}$

D. $\frac{1}{\pi}$

**Answer: A**

**Solution:**

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$$\text{Let } I=\int_{-2}^{2}|x\cos\pi x|dx$$

$$\Rightarrow I=2\int_{0}^{2}|x\cos\pi x|dx$$

$$\Rightarrow \quad I=2\left[\int_0^{1/2}x\cos\pi x dx-\int_{1/2}^{3/2}x\cos\pi x dx\right.$$

$$\left.+\int_{3/2}^{2}x\cos\pi x dx\right]$$

$$\Rightarrow I=2\left[\left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_0^{1/2}-\left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_{1/2}^{3/2}\right.$$

$$\Rightarrow \left.+\left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_{3/2}^{2}\right]$$

$$I=2\left[\left(\frac{1}{2\pi}+0\right)-\left(0+\frac{1}{\pi^2}\right)-\left\{\left(\frac{-3}{2\pi}+0\right)-\left(\frac{1}{2\pi}+0\right)\right\}\right.$$

$$\Rightarrow I=2\left[\frac{1}{2\pi}-\frac{1}{\pi^2}+\frac{3}{2\pi}+\frac{1}{2\pi}+\frac{1}{\pi^2}+\frac{3}{2\pi}\right]$$

$$\Rightarrow I=\frac{8}{\pi}$$

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# Question25

$$\int\limits_{-\pi/2}^{\pi/2}\frac{dx}{e^{\sin x}+1}$$

**KCET 2017**

**Options:**

A. 1

B. 0

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{2}$

**Answer: C**

## Solution:

Let $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x}+1}$ ...(i)

$$\Rightarrow \quad I = \int_{-\pi/2}^{\pi/2} \frac{dx}{e^{-\sin x}+1}$$

$$\left[\because \int_a^b f(a+b-x)dx = \int_a^b f(x)dx\right]$$

$$= \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{e^{\sin x}+1} dx \quad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1dx$$

$$\Rightarrow \quad 2I = [x]_{-\pi/2}^{\pi/2}$$

$$\Rightarrow \quad 2I = \pi$$

$$\therefore \quad I = \frac{\pi}{2}$$

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# Question26

$\int_{-5}^{5} |x+2|dx$ **is equal to**

## KCET 2017

**Options:**

A. 28

B. 30

C. 29

D. 27

**Answer: C**

## Solution:

$$\text{Let } I = \int_{-5}^{5} |x+2|dx$$

$$= \int_{-5}^{-2} |x+2|dx + \int_{-2}^{5} |x+2|dx$$

$$= -\int_{-5}^{-2} (x+2)dx + \int_{-2}^{5} (x+2)dx$$

$$= -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[\left(\frac{4}{2} - 4\right) - \left(\frac{25}{2} - 10\right)\right] + \left[\left(\frac{25}{2} + 10\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= -\left[-2-\frac{5}{2}\right]+\left[\frac{45}{2}+2\right]$$
$$= 2+\frac{5}{2}+\frac{45}{2}+2$$
$$= 29$$

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# Question27

$$\int_{0}^{\pi/2} \frac{1}{a^2 \cdot \sin^2 x + b^2 \cdot \cos^2 x} dx$$

### KCET 2017

**Options:**

A. $\frac{\pi}{2ab}$

B. $\frac{\pi b}{4a}$

C. $\frac{\pi a}{2b}$

D. $\frac{\pi a}{4b}$

**Answer: A**

## Solution:

$$I = \int_0^{\pi/2} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$
$$= \int_0^{\pi/2} \frac{\sec^2 x}{a^2 \tan^2 x + b^2} dx$$

Put $\tan x = t$

$\Rightarrow \quad \sec^2 x dx = dt$

When $x = 0, t = \tan 0 = 0$

and $x = \frac{\pi}{2}, t = \tan\frac{\pi}{2} = \infty$

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# Question28

$\int_{0.2}^{3.5} [x] dx$ **is equal to :**

### KCET 2017

**Options:**

A. 3.5

B. 4

C. 4.5

D. 3

**Answer: C**

## Solution:

$$\text{Let } I=\int_{0.2}^{3.5}[x]dx$$
$$=\int_{0.2}^{1}[x]dx+\int_{1}^{2}[x]dx+\int_{2}^{3}[x]dx+\int_{3}^{3.5}[x]dx$$
$$=\int_{0.2}^{1}0dx+\int_{1}^{2}1\cdot dx+\int_{2}^{3}2\cdot dx+\int_{3}^{3.5}3dx$$
$$=0+[x]_1^2+[2x]_2^3+[3x]_3^{3.5}$$
$$=(2-1)+(6-4)+(10.5-9)$$
$$=1+2+1.5$$
$$=4.5$$

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# Question29

$\int\limits_0^{\pi/2}\frac{\tan^7 x}{\cot^7 x+\tan^7 x}dx$ **is equal to**

## KCET 2017

**Options:**

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

**Answer: A**

## Solution:

$$\text{Let } I=\int_0^{\pi/2}\frac{\tan^7 x}{\cot^7 x+\tan^7 x}dx \qquad \ldots(i)$$
$$\Rightarrow I=\int_0^{\pi/2}\frac{\tan^7(\pi/2-x)}{\cot^7(\pi/2-x)+\tan^7(\pi/2-x)}dx$$
$$\Rightarrow I=\int_0^{\pi/2}\frac{\cot^7 x}{\tan^7 x+\cot^7 x}dx \qquad \ldots(ii)$$
On adding Eqs. (i) and (ii), we get
$$2I=\int_0^{\pi/2}\frac{\tan^7 x+\cot^7 x}{\tan^7 x+\cot^7 x}dx$$
$$=\int_0^{\pi/2}1dx=[x]_0^{\pi/2}=\frac{\pi}{2}$$
$$\therefore\quad I=\frac{\pi}{4}$$